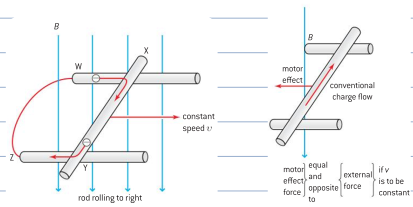
# 11.1 Electromagnetic Induction

- Equations
  - Flux: $\Phi = BA\cos\theta$ or $\Phi = BA$ — $F = BqV\sin\theta$
  - Faraday's / Neumann's equation: $\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$
  - Emf induced in moving rod: $\mathcal{E} = BvL$
  - In wire of coil with N turns: $\mathcal{E} = BvlN$ — $\mathcal{E} = \frac{B\Delta A}{\Delta t} = \frac{NB\Delta A}{\Delta t} = \frac{\Delta(N\Phi)}{\Delta t}$
- Inducing an emf
  - If an electric charge moves through a magnetic field, then a force acts on the charge.
  - A movement or change in a magnetic field relative to a stationary charge gives an electric current.
    - This essentially means that an EMF (and therefore a current) is produced due to a change in the magnetic field.
    - This is known as Electromagnetic induction.
  - The left hand rule can be used for electron flow, but the current must be in the opposite direction because its for conventional current.
  - For an experiment where a magnet moves through a coil or solenoid of wires, a number of conclusions can be made with simple experimentations:
    - The current only appears when there is relative motion between the coil and the magnet (one of the two has to be moving for a current to occur).
      - It doesn't matter if its the coil or magnet that's moving.
      - If both the magnet and coil are moving, then there is no current.
    - When the north end of the magnet is inserted into the coil, the coil will tend to reduce the movement of the magnet by producing another north pole at the magnet end of the coil. 
      - The same will occur with a south pole.
      - The system seems to oppose any change in the magnetic flux (the total amount of magnetic field passing through an area).
      - The greater the rate of change the greater the opposition.
    - When a pole is moved away from the coil, the opposite pole is produced to keep the pole from leaving. 
    - Moving the coil at greater speeds relative to the magnet increases the size of the current.
      - The max is when the axis of the magnet between its poles is perpendicular to the area of cross-section of coil.
  - The figure shows electrons moving through a rod.
    - The electrons and the rod feel a vertical velocity due to the magnetic field going into the page.
    - The electrons have a current to the right.
    - To determine the motion of the conductor, Fleming's left hand rule is used, where the middle finger is velocity of the conductor, index = magnetic field, and thumb = current.
  - Figure 6 shows that the electrons have accumulated to the right of the conductor (rod), leading to a positive charge on the left.
    - This will mean that there will be a pd across the conductor.
    - When there is no external connector between the left and right side of the rod, there will be no current that circulates.
      - Without a current in a closed circuit no work is required, no transformation of energy takes place.
  - If the circuit is closed externally between L and R, then there will be a flow of electrons from the right (high potential) to the left (lower potential). (Figure C)
    - A current has been created or induced.
    - The system is acting to move the electrons through the resistor and, because this is a transformation of energy into an electric form, the source is emf.
    - The reason that the emf in this system is induced is because of the fact that electromagnetic induction is when a conductor is put in an magnetic field where the magnetic field varies or the magnetic field is constant and the conductor moves, creating an emf as the flux is "cut".
      - Electromagnetic induction occurs when the flux lines are "cut" by a conductor (e.g a wire).
        - Flux is the amount of magnetic field in an area.
        - Flux density is the space between the magnetic field lines
        - Flux linkage $= N\Phi$ $= NBA$ (weber turns), this is used to see how much voltage a real generator produces.
- Lenz's law (Any induced current will be in a direction that opposes the change causing it)
  - Induced emf exists whether the charge flows in a complete circuit or not.
  - For figure 6, there is a current as the electrons until an excess of them form an electric field which repel other electrons, stopping further charge movement.
    - When the circuit is completed the charge will flow.
    - An induced emf is always generated by the system and the induced current will only if there is a complete circuit.
  - Fleming's right hand rule can be used to determine the motion of the current and that of the conductor.
    - middle finger = induced current (conventional current), thumb = motion of conductor, and magnetic field = index finger.
  - Lenz's law states: the direction of the induced current is such as to oppose the change of that created the current.
    - Essentially Lenz's law is nothing more than the conservation of energy.

- If the change were to enhance the induced emf (rather than oppose it), this would result in an attraction instead of a repulsion between the magnet and coil.
  - This attraction would mean that the magnet would be pulled towards the coil, accelerating as it goes.
  - The increase in speed which would lead to an even greater acceleration as the motion of the conductor increases.
    - As the magnet moves faster into the coil, gaining kinetic energy from nowhere.
      - which is impossible.
- Another way to look at it is that work can't be done without having some opposition.
  - The induced current in the coil is such that the induced field produced by the current opposes the motion of the magnet.
  - If there is no circuit, then there is no current, no opposition, and no electric energy produced.
  - If you move the magnet very quickly, you will feel the opposite force acting on it.
- In electromagnetic induction, a current is produced due to the motion of the conductor.




When a conductor moves into a uniform field, and a north pole is formed, the current will be in the clock wise, while when its leaving it will be in the anti-clock wise direction.
- This is because of the fact that

- The diagram shows a magnetic rod passing through field lines at a constant speed.
  - The rails conduct and form part a complete circuit (Points WXYZ).
  - Charges, driven by the induced emf, flow around the circuit giving the induced current in the direction shown.
    - The current interacts with the field to give a force (the motor effect force. The motor effect is when a current carrying wire in the presence of a magnetic field experiences a force).
  - Fleming's left hand rule shows that the induced current leads to a motor effect (force) acting on the left of figure 4.
    - This force in the opposite direction of which the conductor is moving, this is what keeps the conductor at a steady speed.
      - This is where conservation of energy comes in.
      - The work done by the external force to keep the conductor moving at a constant speed appears as electrical energy in the conductor.
- Summing up (provided by Bosco Petruccis)
  - Lenz's law states that the direction of the current induced in the conductor by a changing magnetic field (or moving conductor) is such that the magnetic field created (or force created) by the induced current opposes the initial changing magnetic field (or moving conductor).
    - In other words, what ever induces the current in the conductor will be opposed by the induced current.
- Magnetic flux and flux density
  - Fleming's left hand rule shows that the magnetic force arising from the induced current opposes the original force.
    - In other words, the opposing magnetic force is to the left of the original applied force is to the right.
    - The net force is zero, and the rod moves at a constant speed.
  - Work can be done due to the opposition to the motion (created by the external magnetic field that produces a magnetic force on the induced current).
    - If there were no opposition, no work would be possible.
  - To keep the rod (figure 4) at constant speed, a constant force equal to $BIL$ must act on the rod to the right.
    - The energy we have to supply in time $\Delta t$ is: force × distance moved → $BIL \times \Delta x$.
      - $\Delta x$ = distance moved by rod.
  - The induced emf is equal to the energy per coulomb supplied to the system.
    - $\frac{\text{energy supplied}}{\text{charge moved}} = \mathcal{E} = \frac{BIL\Delta x}{Q} = \frac{BIL\Delta x}{I\Delta t}$ therefore $\mathcal{E} = BLv$.
      - Where $v$ is the speed of the rod.
    - $\mathcal{E} = \frac{B\Delta A}{\Delta t}$ = $B \times$ rate of change of area
      - Induced emf = magnetic flux density × rate of change of area.
  - Magnetic flux density is numerically equivalent to the magnetic field strength.
    - When lines of force (field lines) are close together, then the magnetic field strength is large.
      - With many lines through a given area.
      - Magnetic flux density is large when this occurs.
    - The total number of lines per square meter is a measure of magnetic flux density, its the magnitude (strength) of a magnet, electric, or other flux passing through a unit area.
      - While magnetic flux is the total number of lines in a given area.

- A way to think about flux is by imagining a windsock, the stronger the wind, the higher the flux density.
  - The flux is the number of streamlines going through the sock.
  - If the wind has the same speed for two wind socks of different sizes then the larger windsock will have a larger flux even if flux density (magnitude of wind) is the same.
- Flux can be written as: $\Phi = BA$
  - B: Flux density
  - This equation assumes that B and A are at right angles.
    - Therefore, $\Phi = BA\cos\theta$ is used.

- To sum up:
  - magnetic flux density "B" is related to the number of field lines per unit area.
    - It's a vector quantity.
  - magnetic flux is equal to BA ($\Phi = BA$).
    - It's a scalar quantity.
  - the equation $\Phi = BA\cos\theta$ is used if the area is not normal to the lines.
- The unit of flux is weber (Wb) and is defined in terms of the emf induced when a magnetic field changes.
  - The equation $\varepsilon = \frac{B\Delta A}{\Delta t}$ can be re-written as $\varepsilon = \frac{\Delta(BA)}{\Delta t}$ so:
    - a rate of change of flux of one weber per second induces an emf of one volts across a conductor.
- The magnetic flux density $\left(\frac{\text{flux}}{\text{area over which it acts}}\right)$, measured in weber $m^{-2}$) is numerically equal to the magnetic field strength $\left(\frac{\text{force}}{\text{current} \cdot \text{length}}\right)$.
  - One Tesla (T) ≡ One weber per square metre ($Wb\,m^{-2}$).

- Summing up
  - magnetic flux is the amount of magnetic field lines in a coil.
  - Magnetic flux density is the amount of magnetic field lines in one meter squared.
    - It is also numerically the same as magnetic field strength (B).
  - Equations for emf are:
    - $\varepsilon = BLv$, $\varepsilon = \frac{B\Delta A}{\Delta t}$
  - Flux equations:
    - $\Phi = BA$

- Magnetic flux linkage
  - $\varepsilon = \frac{B\Delta A}{\Delta t}$ was used for a single rod.
    - The single rod is equivalent to a single rectangular coil of wire that is increasing in area.
  - The emf across the ends of the coil will be equal to the rate of change of area multiplied by the magnetic flux density.
    - If there are N turns of wire in the coil then the induced emf will be N times greater.
      - $\varepsilon = \frac{NB\Delta A}{\Delta t} = \frac{\Delta(N\Phi)}{\Delta t}$.
        - $N\Phi$ is the magnetic flux linkage.
          - Flux linkage is also written as weber turns.
  - Magnetic induction is summed up in Faraday's law:
    - The induced emf in a circuit is equal to the rate of change of magnetic flux linkage throughout the circuit.
      - $\varepsilon = -\frac{N\Delta\Phi}{\Delta t}$
      - The negative sign is added to include Lenz's law.

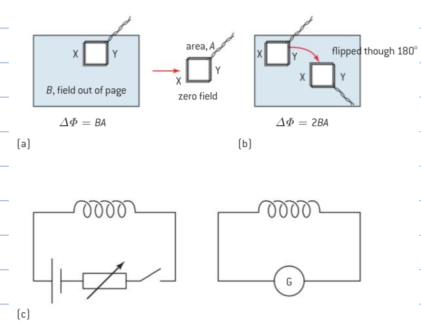


Figure 6 Moving coils and changing fields.

- Changing fields and moving coils
  - Emf can be induced by:
    - A wire or coil moves in unchanging magnetic field.
      - Rolling rod example
    - The magnetic field changes in strength but the conductor doesn't move.
    - A coil changing in size or orientation in an unchanging magnetic field.
      - $\varepsilon = BvLN$ or $\varepsilon = -\frac{N\Delta\Phi}{\Delta t}$, and $\varepsilon = BA\cos\theta$
      - Increasing the coil number and length will increase the flux linkage, and changing the orientation will change the flux.
    - Combinations of these changes can occur.
  - Case 1: Straight wires moving in uniform field.
    - This is the case of the rolling rod.

- The change in the area is $lv$, $l$ = length of rod and $v$ is the speed of rod.
- The induced emf is therefore $\varepsilon = Bvl$ when the wire moves at 90° to the field lines.

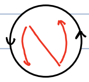
- Case 2: Coil moving
  - The coil can move as shown in figure 6(a) from one position in a magnetic field to another position where the field may be different.
  - If the coil begins and ends in positions where the field is identical, then there is no change in the flux linkage ($\Phi N$) and there is no induced emf.
    - Even though the coil is cutting the field lines, the same number is being cut on opposing sides of the coil.
    - Two emfs are induced but in the opposing senses and cancel out.
  - If the coil moves from a place where the flux is $\Phi$ to a position where the flux is zero, the change in flux linkage is $N\Phi$ and the induced emf is $\varepsilon = \frac{N\Phi}{\Delta t}$.
  - When a coil in a field is flipped through 180° (figure 8).
    - The field lines seem to reverse their direction through the coil, so the change in flux is $2\Phi$ ($\Phi - (-\Phi)$).
      - $\varepsilon = \frac{2N\Phi}{\Delta t}$
    - When a coil rotates in a field the emf produced instantaneously depends on the rate of change of the flux linkage, and therefore depends on the angle.
      - If the coil rotates at a constant angular speed, then the emf will vary sinusoidally.
        - This is the basis of an alternating current.

- Case 3: Magnetic field changes
  - Sometimes the field changes from one value to another.
    - It gets stronger or weaker, but the coil doesn't move.
  - Suppose the field is being turned on from zero.
    - Before the field changes, there are no field lines in the coil.
  - $\varepsilon = -\frac{N\Delta\Phi}{\Delta t}$ becomes $\varepsilon = -\frac{NA\Delta B}{\Delta t}$ since only B changes.

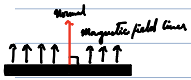
- Worked example
  - $\varepsilon = \frac{N\Delta\Phi}{\Delta t}$
    $= \frac{(500)(2\cdot10^{-3})}{4\cdot10^{-3}}$
    $= 250$ Volts
  - $L = 1.5m$, $\Delta t = 0.5s$
    - $s = ut + \frac{1}{2}at^2$
      $1.5 = \frac{1}{2}(9.81)t^2$
      $t = 0.55$ seconds

  - The reason being that Lenz's law states that the induced current in the conductor will create an opposite force in the movement of the conductor.
    - This will mean that the conductor will be moving at a constant speed.
    - The aluminium won't experience this because of the fact that it doesn't have an induced emf as it's not a conductor.

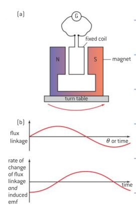
- 11.2 Power generation and Transmission
  - Equations
    - rms and peak values: $I_{r.m.s} = \frac{I_0}{\sqrt{2}}$
    - Potential difference: $V_{r.m.s} = \frac{V_0}{\sqrt{2}}$
    - Resistance: $R = \frac{V_0}{I_0} = \frac{V_{r.m.s}}{I_{r.m.s}}$
    - maximum Power: $P_{max} = I_0 V_0$
    - Average Power: $P_{average} = \frac{1}{2} I_0 V_0$
    - transformer equation: $\frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}$
  - Alternating current (ac) generators
    - AC generators are commonly used for generation of energy.
      - Such a generator consists of a coil with a large number of turns.
      - The coil rotates relative to the magnetic field.
    - An example is the following:
      - A coil is fixed between the poles of a U-shaped magnet that stands at the center of a rotating turntable.
      - The turntable can turn at different angular speeds and the coil can have different numbers of turns and cross-sectional areas.
      - When the magnetic flux in the coil is maximum, the emf induced (current) is minimum (0) and vice-versa.
      - Changing the speed of the table changes the frequency of the emf as well as its amplitude.
      - Increasing the coil numbers or the area of the coil will increase the amplitude of the emf but leave the frequency unchanged if the turn table speed doesn't change.
      - Overall: $\varepsilon = -N\dot{\phi} = -NBA\cos\theta$ $\quad \varepsilon = -BA(\Delta\cos(\omega t)) = BA\omega\sin(\omega t)$ $\quad \varepsilon = BAN\omega$ $\quad P = \varepsilon^2 = V_r^2 \div R$ $\quad V_{RMS}^2 = V_r^2 \rightarrow V_{RMS} = \sqrt{V_r^2}$ $\quad V = V_r$



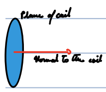
- If a fixed coil is placed between a ω-shaped magnet on a turning table.
- When the magnetic flux (the amount of magnetic field lines that is in the coil) is at a maximum, then the induced emf (current) will be at a minimum (0).
  - When the magnetic flux is at a minimum the emf is at a maximum.
  - This is because of the fact that according to Faraday's law: $-\frac{N\Delta\phi}{\Delta t}$, when the rate of change is greatest, the emf will be largest. This occurs when flux linkage is 0.
- Changing the speed of the turntable will change the frequency of the emf as well as its amplitude
- Magnetic field lines cut the coil as the turntable rotates and an induced emf is generated.
  - The emf will vary as the turntable rotates.

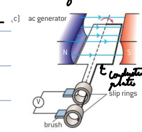



- As seen in the table, the flux linkage varies with θ, the angle between the normal.
  - When θ is equal to 90°, the field lines lie in the plane of the coil and so the flux through the coil is zero ($\cos 90° = 0$ in the equation $\phi = BA\cos\theta$).
  - When θ is equal to 0°, the field lines are perpendicular (90°) to the plane of the coil and the flux through the coil is a maximum.
    - What is meant by these angles is that when the magnetic field is in the same direction as the normal of the coil (or conductor), then the angle will be zero, meaning the flux will just be equal to $\phi = BA$.
    - If instead the magnetic field is at 90° to the coil's normal, the angle would be $\cos 90° = 0$, resulting in no flux.
  - If the rotation speed (ω) is constant, the flux will vary as a sine graph.




- The emf induced in the coil is equal to $-\frac{\Delta\phi}{\Delta t}$, which is the negative gradient of the flux-time graph.
- While some ac generators have rotating magnets, and a fixed coil, others have a fixed magnet, and a rotating coil.




- 


The conducting plate rotates in a uniform field, resulting in the normal of the plate cutting and being in parallel with the magnetic field.

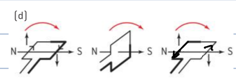
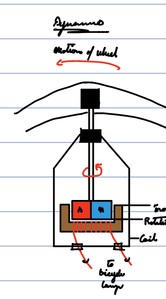
- 


Figure 1 Basic ac generator.

- When the wire is moving (left side moving up, right side down) then the current induced is that it will move from the North pole to the south pole, when the coil completes a half revolution (the left side moves to the right) then the current will flip into the opposite direction.
- The essential requirements for an ac generator are therefore:
  - A rotating coil
  - A magnetic field
  - Relative motion between the coil and magnetic field
  - Suitable connections to the outside world.




- An issue will arise with a real life ac generator.
  - In subtopic 11.1 any moving conductor carrying an induced current in a magnetic field will have two forces acting on it:
    - Force 1 is the force that moves it.
    - Force 2 is the opposite force that arises because of the induced current.
      - known as Lenz's Law.
  - This also applies to the rotating coil in the ac generator.
    - Fleming's left hand rule and Lenz's law show that this force opposes whatever is turning the coil.
      - So if an applied force is moving the coil, then an opposite force will occur.
      - If a generator coil is being turned clockwise by an external agent, then the induced magnetic force (induced by induced current interacting with magnetic field) will exert a turning force in the opposite direction.
        - This will reduce the induced current that can be made available to the circuit.
          - Although, if this weren't the case then the law of conservation of energy would broken.
- An example of a bicycle dynamo, where a rotating device is placed on the wheel to spin a permanent magnet between a coil.
  - The opposite turning force will be experienced by the biker as like and opposite poles are produced at the opposite ends of the magnet to keep the amount of flux constant.
    - This is Lenz's Law
- Nature of Science

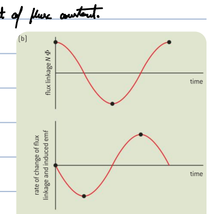
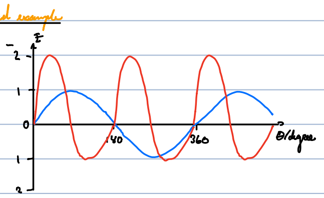



Figure 3

  - The flux linkage ($\omega NBA\cos\theta$) time graph will have a cosine shape with a max and min value of $NBA$ at $t=0$ and $-NBA$ at halfway of one cycle.
  - The value of the induced emf is equal to the negative rate of change of the flux linkage ($N\phi$). $-\frac{N\Delta\phi}{\Delta t} = \varepsilon$
    - Which is equal to the negative gradient of the flux linkage-time graph.
    - Which is a negative sine curve.

- max and min are $\varepsilon_0 = \pm BAN\omega$.

- A supply where the current and voltage (emf) vary as a sine wave is an alternating supply.
- If the angular speed (speed at which the coil moves) is increased but keeping other features the same, then:
  - The coil will take a shorter time to complete one cycle, meaning that there will be more cycles per second, and a higher frequency.
  - The time between max and min flux linkage will decrease and therefore (flux linkage is the same) the peak EMF will increase.
- To increase the emf without changing the frequency the following can be change:
  - Magnetic field strength (B)
  - number of turns in the coil (N)
  - The area of the coil (A).
  - This is because of the fact that $\varepsilon = \omega NBA$.
- Worked example




The reason that the frequency would increase is because of the fact that while the flux linkage remains the same, it will take half the time for the coil to complete a full rotation. This will increase the frequency. Since emf is given by $\varepsilon = \omega NBA$, increasing rotational speed increases EMF.

- Measuring alternating currents and voltages
  - The current and the emf (voltage) alternate constantly in one cycle.
  - One way to measure the current and voltage is by considering the power supplied to the resistor.
    - $P = I^2R$, where $I$ is the instantaneous current.
  - As seen by the two graphs (graph 1 being the current variation in one oscillation, and the other being the power in the resistor at a certain time), the following differences are notable:
    - The power-time graph is always positive, because even if the current is negative the formula $P = I^2R$ will make it positive.
    - The power graph has double the frequency of the current.

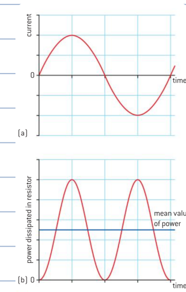


Figure 5 Current–time and power–time graphs for ac.

  - If a lamp is supplied with an ac supply, it will turn on and off twice in one cycle.
    - The lamp is on when the emf ($\varepsilon = \sqrt{PR}$) is at a max (positive value) or minimum.
    - We don't notice this because since the frequency of the light bulb is 50 or 60Hz, it will turn on and off at 100 or 120 times per second.
  - A lamp supplied with a dc supply will have the same brightness as the average brightness of an ac lamp.
    - Since the current in ac generator goes from positive to negative, the squared value of the current will be used to calculate the average of the power.
      - The current for the circuit will be given by the equation $I_{RMS} = \sqrt{\frac{I^2_{max}}{2}}$
      - The voltage will be given by: $V_{RMS} = \sqrt{\frac{V^2_{max}}{2}}$
      - The power would be given as: $P = I_{RMS} \times V_{RMS}$.
  - Summing up
    - Overall, in an alternating current, since the current will move from the positive axis to the negative one, to measure the average, and the power, the squared value of the current will have to be used.
      - This is so that the average won't be zero.
      - The formula will be: $I_{RMS} = \sqrt{\frac{I_{max}^2}{2}}$
        - RMS = Root mean squared
        - $I_{max}$ is the peak value of the current.
    - The power will always be positive due to the fact that the current and voltage even though they are negative at times they will be positive due to the fact that their values are squared.
      - $V_{RMS} = \sqrt{\frac{V^2_{max}}{2}}$
    - The resistor is constant.
    - Power calculated with RMS values gives the average power.
    - The frequency for the power graphs is twice that of the current.
      - This is due to the fact that the value of the currency is squared
- Worked example

- $\mathcal{E}_{RMS} = \sqrt{\frac{360^2}{2}}$ $= 255\,V$
- $F = \frac{1}{T}$ $= \frac{1}{0.02}$ $= 50\,Hz$
- $P_{average} = I_{RMS} V_{RMS}$ $= \sqrt{\frac{9.8^2}{2}} \times \sqrt{\frac{140^2}{2}}$ $= 665\,W$ $\approx 670\,W$

- **Transformers**

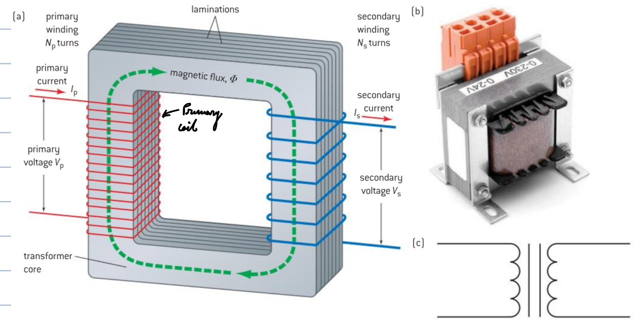


Figure 6 Transformers in theory and practice.

- Transformers can be used to change alternating supplies from one pd to another.
- A Transformer consists of:
  - An input (or primary) coil
  - An output (or secondary) coil
  - An iron core on which coils are wound
- The Transformer like an ac generator relies on electromagnetic induction for it to work.
  - An alternating current is supplied to the primary coil.
    - A magnetic field is produced by the current in the primary coil and this field links around the core (the metal).
  - The magnetic field in the core alternates due to the alternating current.
    - It alternates at the same frequency.
  - The changing flux links to the secondary coil.
  - Due to the fact that the alternating current (input one) will induce an alternating magnetic field in the iron core, a secondary induced current (which alternates due to the alternating magnetic field) will be outputted.
    - When a resistor is connected to the secondary coil, a circuit will be formed with the secondary coil and the load (resistor).
  - Energy has been transferred from the primary to the secondary circuit through the core.
- If an alternating pd with a peak value of $V_p$ is applied to the primary coil it will result in a flux $\Phi$ in the core.
  - The flux will be linked to the secondary coil by the iron core.
  - The secondary coil with $N_s$ turns will have a linked flux of $N_s\Phi$ and an induced emf in the secondary coil of $V_s = N_s \frac{\Delta\Phi}{\Delta t}$.
  - There is also a flux linkage to the primary coil even though the primary current was responsible for the field
    - Giving rise to $\mathcal{E}_p = N_p \frac{\Delta\Phi}{\Delta t}$.
    - The induced primary emf will oppose the applied pd. Ask if the induced emf is the one for the primary current, or for the secondary current.
- If the resistance is negligible for the primary coil, then the $\mathcal{E}_p$ will be equal to $V_p$.
  - Therefore, since $\frac{\Delta\Phi}{\Delta t}$ is the same for both coils, the equations are:
    - $\frac{\mathcal{E}_p}{N_p} = \frac{V_p}{N_p} = \frac{V_s}{N_s} \longrightarrow \frac{\mathcal{E}_p}{\mathcal{E}_s} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$.

    - This is known as the Transformer rule:
      - When $N_s > N_p$ then that's called a step-up transformer
        - The larger amount of coils for the secondary output means that the output emf will be higher.
      - When $N_s < N_p$ then that's called a step-down transformer.
        - The lower amount of coils will mean that the output emf is lower
      - Step-down and step-up refers to the emf rather than the current.
- When there is zero current in the primary coil, then there will be no energy transfer in the second one.
  - The equation only applies when the current in the secondary coil is zero (when nothing has been connected yet).
- Lenz's law applies here as well when there is a current in the secondary current.
  - A secondary magnetic field set up through the core due to the secondary current.
  - The magnetic field tries to oppose the changes occurring in the system, reducing the flux in the core.
    - This will lead to the emf decreasing in the primary coil.
    - This will mean that there is now an overall current in the primary that allows energy to be transferred. ?
- We assume no energy lost Therefore, $I_p V_p = I_s V_s$.
  - The efficiency is given by:
    - $\frac{\text{energy supplied by secondary coil}}{\text{energy supplied to primary coil}} \times 100$ ⟶ $\text{Efficiency} = \frac{P_2}{P_1} \times 100$ ⟶ $\text{Efficiency} = \frac{\mathcal{E}_2 I_2}{\mathcal{E}_1 I_1} \times 100$

  - To improve the efficiency:
    - Laminating the core: How do they work? Glue multiple layers of iron sheets together
      - Iron is a good conductor and the changing flux in the transformer produces currents flowing inside the iron core.

- Known as eddy currents.
    - Eddy currents are the currents produced when a conductor moves through a magnetic field.
    - The currents are formed when a magnet that is opposing the change in flux, their direction can be determined with the right hand rule.
  - To prevent eddy currents from being created, thin layers of insulating material are placed between the sheets or iron in the core.
    - This will mean that while the magnetic properties of iron won't be altered, the resistance in the core will increase significantly.
    - The currents are forced to travel along longer paths within the layers, increasing electrical resistance and reducing the current.
    - Laminations reduce the energy losses that result from a reduction in the amount of flux and from a rise of temperature of the iron that would occur if the eddy currents were large.
- Choosing the core material:
  - The magnetic material of the core is a "soft" magnetic magnetic.
    - "Soft" magnetic materials can be magnetised and de-magnetised quickly and easily.
- Choosing the wire in the coils
  - Low-resistance wires are used in the primary and secondary coils.
    - High resistance wires would lead to heating losses (called joule heating) in the coils.
- Core design
  - It's important to not allow flux to leak out of the core.
  - As much flux should be linked between both coils so that the max rate of change of flux linkage occurs.

- Worked example
  - $\mathcal{E}_1 = 120V$, $\mathcal{E}_2 = 5V$, $n_1 = 2300$
    $\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{n_1}{n_2}$
    $n_2 = \frac{n_1 \mathcal{E}_2}{\mathcal{E}_1}$
    $n_2 = \frac{(2300)(5)}{120}$
    $n = 96$ turns
  - $P = VI$
    $I = 0.2$ Amps
  - $\frac{P_2}{P_1} \times 100 =$ Efficiency
    $= \frac{0.2 \times 5}{0.009 \times 120} \times 100$
    $= 92.6\%$

- Transformers in action
  - To help with minimising energy losses, energy is sent at high voltages and low currents.
  -

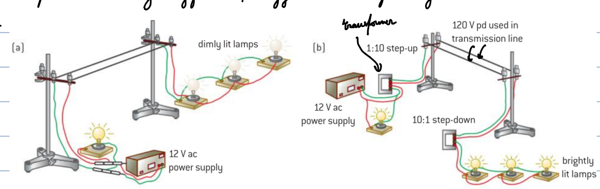
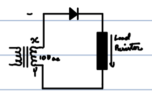


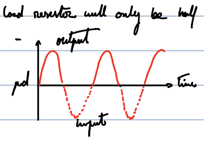
Figure 8 Transmission at high voltages.

  - The image shows two different methods of transmitting electrical energy from one place to another.
    - As seen, there are two generators that provide energy at an alternating potential difference of 12V.
    - This energy has been transmitted through two transmission lines
    - The step up transformer with a turns ratio of 1:10 is used to transmit the energy at 120V.
      - Multiplies value by 10.
      - At the other end of the transmission line is a step-down transformer which brings back the voltage to the required amount by the lamps in the circuit.
  - For both of the transmission lines, the lamps are rated at 0.3A, 12V, power requirement of 3.6 Watts, and total resistance of the transmission line being 1.5 ohms.
    - The total current required will be 0.9A, with the power lost in the transmission lines being $P = I^2R = 0.81 \times 1.5 = 1.2W$
    - For the second transmission line, the voltage is 120V, the current will be stepped down to 0.09A. $I = \frac{P}{V}$
      - This will make the power loss of the transmission line $0.081 \times 1.5 = 0.012W$.
      - Stepping the voltage up by 10 will mean that the current will decrease by 10, this will reduce power lost by 100 ($10^2$).

- Rectifying ac
  - Converting an ac current to a dc current it's called rectification.
    - A device that does this is called a rectifier.
      - There are two rectifier kinds, half wave and full wave.
        - Half wave is where only half of each cycle of the current is used.
        - Full wave is when both of the halves of the ac cycle are used.
  - Half-wave rectification
    - The figure shows the basic circuit.

- 


- The image shows a single diode connected in series with the secondary terminals of a transformer and the load resistor.
- The load resistor represents the part of the circuit that is being supplied with the rectified current.
- Diodes are devices which only allow charge to flow through them in one direction.
    - The symbol is an arrow head which points in the direction of the flow of the conventional current allowed by the device.
        - It's said that the diode is forward biased when it's conducting.
        - When there is no current, then it's said that the diode is reverse biased.

- The load resistor will only be half a cycle.
    - 


    - The wavefront isn't exactly half a cycle wide because of the fact that it requires a small forward pd for conduction to begin.
    - When terminal x is positive, and terminal y is negative, then there will be a current through the diode in the forward direction.
        - This is with the use of conventional current.
    - When the terminal x is negative, and terminal y is positive there will be no current in the circuit as the diode will prevent the flow of current.

- While a diode will keep the direction of the current in one direction, then the current won't be be constant.
    - The wavefronts need to be closer so that they are more of a dc constant value.
        - To do this, a resistor and capacitor connected in parallel between the diode and load.

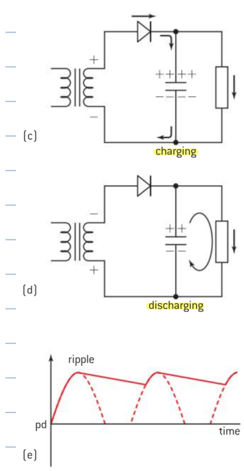
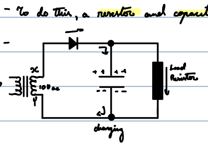
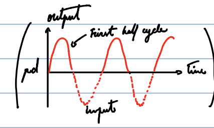
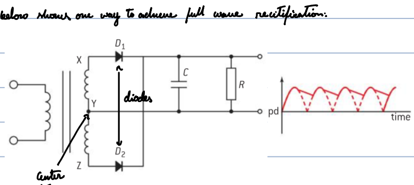


Figure 10 Half-wave rectifier.

- 




- During the first half cycle, the capacitor charges up and the potential difference across it approaches the peak of the transformer output emf.
    - When the current is zero in the second (negative) half-cycle, the capacitor will discharge at a rate at which is determined by the time constant of the circuit.
        - If the time constant is much larger than the time of a cycle, the amount of charge released by the capacitor will be small and the pd will not change very much.
    - When the diode conducts again in the next positive half-cycle, the capacitor will be filled up.
    - This will create a discharge-charge cycle with the pd across the capacitor varying a lot less.
        - Refer to image "e".
- A large time constant will result in good smoothing, but at a very large cost.
    - Capacitors with a large time constant are called "reservoir" capacitors.
    - Capacitors with a small variation are called "ripple" voltage.

- Full wave rectification
    - Some applications won't be able to operate with with a half-wave rectifier, therefore, full-wave rectifiers are used.
    - The figure below shows one way to achieve full wave rectification.
        - 


    - As can be seen, this arrangement uses two diodes and requires a center-tap on the transformer.
        - This means that there is a connection half way along the length of the wire that has been wound to make a second coil
        - The diagram also shows that there is a resistor-capacitor pair which will smooth the rectified wave.
    - Imagine Y is always at 0 potential.

- Then, half the time $x$ will be positive, and $z$ will be negative relative to Y.
  - The other half, they reverse.
- When $x$ is positive relative to Y, diode $D_1$ will conduct.
  - Only half of the secondary coil connected to $D_1$ takes part in conduction at any time. ???
    - Are they talking about the $D_1$ & $D_2$ circuits.
  - When $x$ has become negative $D_1$ won't conduct, making $z$ positive relative to Y.
    - $z$ with $D_2$ will conduct for the half a cycle.
    - Current will then be again supplied to a capacitor-resistor combination during both halves of the cycle, leading to full rectification.

-

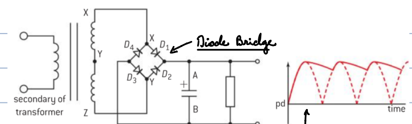


Figure 11 Full-wave rectification.

Higher value than the figure 11 a pd-time graph.

Write notes for this section! ? p.460

- In order to achieve a particular value of peak pd, twice as many turns are required on the secondary compared to the half-wave arrangement.
  - What they're trying to say is that for the peak value of the pd to be maintained, then a significant amount of coils must be used for the secondary because then it won't go down the required pd value.
  - The disadvantage can be overcome with a diode bridge.
- The full secondary coil is now used and the diode arrangement allows the whole of the coil to supply current through the cycle.
  - Disadvantage: Need four diodes, and it's a more complex circuit.
- When $x$ is positive relative to $z$, then the junction between $D_1$ & $D_4$ is positive relative to the junction between $D_2$ & $D_3$.
- $D_1$ is the one that conducts so that point A of the capacitor becomes positive.
  - $D_3$ conducts, and point B of the capacitor becomes negative.
  - The capacitor charges and current is supplied to the rest of the circuit.
- When the polarity of the secondary coil switches, $x$ becomes negative relative to $z$ and the conducting diodes are now $D_2$ & $D_4$.
  - Polarity of capacitor is unchanged.
- These examples are said to be passive.
  - They don't amplify or modify wavefronts.

- Wheatstone and Wien bridge circuits
  - The four-diode rectifier arrangement is one of a class of circuits known as bridge circuits.
  -

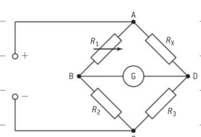


Figure 12 The Wheatstone bridge.

- The Wheatstone bridge is a completely resistive arrangement that can be used to estimate the resistance of an unknown resistor.
  - Normally used in a dc context.
- In the circuit there are three known resistors $R_1$, $R_2$, $R_3$ connected with an unknown resistor $R_x$.
  - One of the known resistors is variable and is adjusted until the current in the galvanometer that "bridges" the pairs of resistors is zero.
- When the current in the galvanometer is 0, the bridge is said to be balanced.
  - This is because of the fact that there is no potential difference between B and D.
  - This means that the pd across $R_1$ & $R_x$ are the same as each other, and the pds across $R_2$ & $R_3$ are also identical.
    - $V_1 = I_1 R_1 = I_2 R_x$   $V_2 = I_1 R_2 = I_2 R_3$ so $\frac{I_1}{I_2} = \frac{R_x}{R_1} = \frac{R_3}{R_2}$ therefore $R_x = \frac{R_1 R_3}{R_2}$.
- The Wien bridge circuit is a modification of the Wheatstone arrangement to allow the identification of resistance and capacitance values of an unknown component.
  - Operates with ac supply.
  - The modification is the addition of a capacitor which is in series with $R_2$.
  - The current in the outer arm is measured.
  - $R_2$, $C_2$ and the frequency of the supply need to be adjusted overall, for min current.

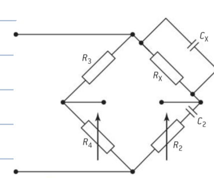
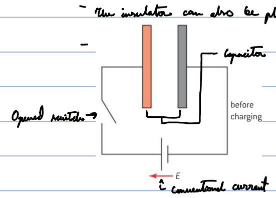


Figure 14 Wien bridge.

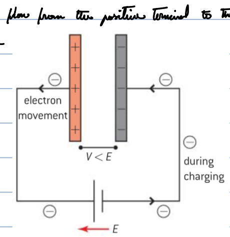
- 11.3 Capacitance
  - Equations:

- Definition of capacitance: $C = \frac{Q}{V}$
- Combining capacitors in parallel: $C_{parallel} = C_1 + C_2 + \dots$
    - In series: $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$
- Capacitance of a parallel-plate capacitor: $C = \varepsilon \frac{A}{d}$
- Energy stored in a capacitor: $E = \frac{1}{2}CV^2$
- Time constant: $\tau = RC$
- Exponential discharge charge: $Q = Q_0 e^{-\frac{t}{\tau}}$
- Current: $I = I_0 e^{-\frac{t}{\tau}}$
- Potential difference: $V = V_0 e^{-\frac{t}{\tau}}$

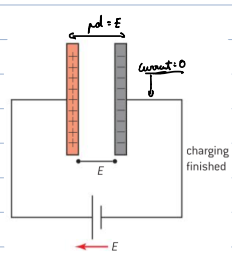
- Capacitors in theory
    - The arrangement of parallel plates in which the two plates are separated by an insulator called a capacitor.
        - The insulator may be a vacuum.
            - It can either be air or another gas, as long as a spark can't jump from one plate to another.
        - The insulator can also be plastic.
        -




- The image shows two parallel plates connected to a cell.
- The plates are initially uncharged.
    - When the switch is closed, electrons begin to flow (since the circuit has been completed).
        - There is no current between the plates because of the insulation.
        - Electrons flow from the positive terminal to the negative one.
        -



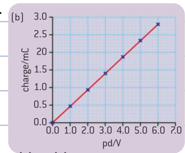

If more electrons are moved to another plate, the pd will increase.
- $U \propto Q$
    - Where U is potential, and Q is charge.




- Charge is being separated by the system and stored.
    - This requires energy (power), which is provided by the cell.
    - Energy is being stored on the plates as the electrons arrive there.
- As more electrons move to the negative plate, more work will have to be done because of the electrostatic repulsion between electrons.
    - Eventually the repulsion will be so strong that there will be insufficient potential energy in the cell to move more electrons.
    - When this happened, the pd of the capacitor will be equal to the pd of the emf.
        - It can't be more than the cell's emf.
- Capacitance, $C = \frac{\text{charge stored on one plate}}{\text{potential difference between plates}} = \frac{Q}{V}$

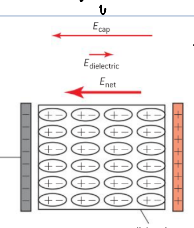
    - Capacitance is the ratio of charge stored to the potential difference.
    - The charge stored on one plate is the same as the charge transferred through the cell and moved from one plate to the other.
    - The units are: Farad (F) or Coulomb per volt ($CV^{-1}$).
        - Fundamental units is $A^2 s^4 kg^{-1} m^{-2}$.

- Worked example
    - $25 \times 10^{-6}$ C, pd = 15V
        - $C = \frac{Q}{V}$
          $= \frac{2.5 \cdot 10^{-6}}{15}$
          $= 1.67 \cdot 10^{-7}$ Farad
          $= 167$ nF
    - $C = 0.15\ \mu F$, $Q = 7.8 \cdot 10^{-8}$ C
        - $V = \frac{Q}{C}$
          $V = 0.52$ V

- Energy stored in a capacitor
  - 

  - The gradient of a pd-charge graph is equal to the capacitance.
  - The area under the graph gives the energy stored.
    - Energy stored on capacitor $= \frac{1}{2}QU = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CU^2$
      - Units = Joules
- Worked example
  - Energy stored $= \frac{1}{2}(25)^2(0.005)$
    $= 1.6J$
  - $C = 100mF$, Energy $= 2505$
    - $U = \sqrt{\frac{500}{0.1}}$
      $= 71U$
- Capacitance of a parallel-plate capacitor
  - For a pair of parallel plates:
    - $\frac{Q}{A} = \varepsilon_0 \frac{U}{d}$
      - Q is the charge stored, A is the area of overlap of the plates, U is the potential difference between the plates, and d is the separation of the plates.
    - Rearranged: $C = \varepsilon_0 \frac{A}{d}$
      - This enables us to calculate the capacitance of a capacitor given the area of overlap and the plates and their separation.
      - In real capacitors, edge effect will reduce the value.
  - If the gap between the plates is filled with an insulator of permittivity $\varepsilon$:
    - $C = \varepsilon \frac{A}{d}$
- Worked example
  - $A = 0.015m^2$, $d = 2 \cdot 10^{-3}m$
    - $C = \frac{8.9 \cdot 10^{-12} \cdot 0.015}{2 \cdot 10^{-3}}$
      $= 6.68 \cdot 10^{-11}$ Farad
- Capacitors in practice
  - Capacitors are used to store charge and reduce the effect of fluctuations in a circuit.
  - The equation $C = \varepsilon_0 \frac{A}{d}$ shows that there are 3 ways to increase the capacitance:
    - Decrease distance between plates.
    - Increase area of the plates.
      - Increasing the area of the plates will increase the value of the capacitance because of the fact that more charge can be stored for a given potential difference between the plates and therefore $\frac{Q}{V}$ increases.
    - Change the constant of permittivity from that of a vacuum to another one.
      - Insert a material between the plates that replaces air or vacuum.
      - $\varepsilon_0$ is changed to $\varepsilon$.

original field
V

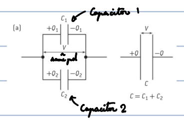


▲ Figure 4 How a dielectric increases capacitance.

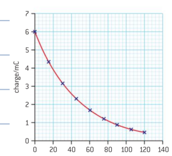
- A dielectric material is an electrical insulator that is polarised when placed in an electric field.
  - The reason that it's polarised is because of the fact that at one end of the molecules it's slightly more positive than the other end.
    - This means that when the material is in an electrical field, the molecules will either move slightly or by rotating the positive end of the molecule in the direction of the field.
  - If the pd across a capacitor is increased, a max field strength will be reached.
    - If it's passed, the material will become a conductor.
    - The max field is called dielectric strength.
- As seen in figure 4, the original field, $E_{cap}$, is from right to left (positive to negative), but the dielectric field is from left to right.
  - Therefore, the net field will be $E_{net} = E_{cap} - E_{dielectric} = \frac{U}{d}$.
    - Since d is constant, U will decrease.
  - The potential decreases because of the fact that work has to be done by the capacitor to align the molecules in the material.
    - Since the potential will decrease as energy in the capacitor is used for work, the overall charge will remain unchanged, $\frac{Q}{V}$, therefore the capacitance will increase.

- The dielectric increases, the capacitance increases as well.

- **Worked example**
    - $A = 100\ cm^2$, $d = 1.5\ cm$
        - $C = \frac{(\varepsilon)(0.01)}{(0.015)}$ $\quad 1.00054 = \frac{\varepsilon}{8.9 \cdot 10^{-12}}$
          $C = 5.94 \cdot 10^{-12}$ Farad $\quad \varepsilon = 8.9 \cdot 10^{-12}$ (air)
        - $C = (8.9 \cdot 10^{-12})(3)$ $\quad C = 1.78 \cdot 10^{-11}$ Farad
          $= 2.67 \cdot 10^{-11}$ (polyester)
    - $C = \varepsilon \frac{A}{d}$
      $A = \frac{Cd}{\varepsilon}$
      $= \frac{(1 \cdot 10^{-9})(0.01)}{8.9 \cdot 10^{-12}}$
      $= 1.12\ m^2$

- **Combining capacitors in parallel and series**
    - Capacitors like resistors can be connected in series and parallel.
        - Connecting them will also modify their value.
    - **Parallel**
        - 

        - When connected in parallel, the potential difference between across the capacitors will be the same.
        - The total charge stored is $Q_1 + Q_2$. The reason that their charges are different is because they have a different
            - $Q_1 = VC_1$, $Q_2 = VC_2$.
        - The single capacitor equal to the two capacitors in parallel will have a charge of $Q = VC$.
            - Therefore, since the potential difference is the same for both the capacitors:
                - $C = C_1 + C_2 \longrightarrow Q = V(C_1 + C_2)$.
        - When two capacitors are connected in parallel, the total capacitance is equal to the sum of the capacitances.
    - **Series**
        - Capacitors in series have the same amount of charge.
            - This is due to the fact that they have the same current that charges both of them.
            - Example of Kirchhoff's first law.
        - The potential differences will add up to give the emf of the cell.
            - Kirchhoff's second law.
            - $V = V_1 + V_2$
        - Capacitance is given by: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$
            - The reciprocal of the total capacitance is equal to the sum of the reciprocals of each capacitance.
    - **Worked example**
        - $C = \frac{1}{C_1} + \frac{1}{C_2}$
          $C = \frac{2}{6}$
          $C = \frac{1}{3} C$

- **Discharging and charging a capacitor**
    - **Discharging**
        - The capacitor can be thought as taking the place of a power source and the current and pd is related by:
            - $V_C = IR$
            - $V_C$ & $I$ change overtime.
        - $\frac{\Delta Q}{\Delta t} = \frac{V_C}{R}$
            - Q is the charge on the capacitor, and t is time.
        - $\Delta Q = \frac{-Q\Delta t}{RC} \longrightarrow -\frac{Q\Delta t}{\tau}$ ($\tau$ is a constant)
            - The equation shows how the charge on the capacitor varies with time during the discharge.
        - 

          ▲ Figure 9 Charge versus time for a discharging capacitor.
            - This is the same shape as the graph of time - pd graph.

- Because, $C = \frac{Q}{V}$ and so $Q \propto V$.

- $\Delta Q = \frac{Q \Delta t}{\tau}$
  - At time $t = 0$ the charge on the capacitor will be $Q_0$.
  - the charge remaining on the capacitor after $\Delta t$ will be $Q_0 - \frac{Q_0 \Delta t}{\tau}$.
  - the charge line has a gradient of $-\frac{Q_0}{\tau}$ during the first time interval $\Delta t$.
  - the gradient is $\frac{Q_1}{\tau}$.
- the charge in a capacitor will also have a half-life similarly to the half-life of a nucleus.
  - therefore, the capacitor will have exponential decay.
  - $\frac{\Delta Q}{Q} = -\frac{\Delta t}{\tau} \equiv \frac{\Delta N}{N} = -\lambda \Delta t$ & $N = N_0 e^{-\lambda t} \equiv Q = Q_0 e^{-\frac{t}{\tau}}$
- the capacitor discharge equation can be written as: $\log_e Q = \log_e Q_0 - \frac{t}{\lambda}$.
  - the gradient is $-\frac{1}{\lambda}$.

- **Worked example**
  - $C = 220\mu F$, $V = 30V$, $R = 330k\Omega$
    - $10 = 30e^{-\frac{t}{72.6}}$
      $t = (72.8)(-\ln e^{\frac{1}{3}})$
      $t = 79.8$ seconds
    - $T = RC$
      $= (330 \times 10^3)(220 \cdot 10^{-6})$
      $= 72.6$
  - $Q = Q_0 e^{-\frac{t}{\tau}}$
    $= (6.6 \cdot 10^{-3}) e^{-\frac{79.8}{72.6}}$
    $Q = 0.0022$
    **$\Delta Q = 4.4mC$**
  - $Q_0 = VC$
    $= (30)(220 \cdot 10^{-6})$
    $= 0.0066$ Farad
  - $0.1 = e^{-\frac{t}{\tau}}$
    $t = 2.3RC$

- **Charging**
  - **A model for charging**
    - the capacitor is in series with a resistor of resistance R and a cell of emf $V_{emf}$.
    - when the circuit is switched on with the capacitor uncharged, charge begins to flow.
      - $V_{emf} = V_C + V_R$ (Kirchoff's first law), therefore, $V_R = V_{emf} - V_C$.
        - so, $I = \frac{(V_{emf} - V_C)}{R}$, $I = \frac{\Delta Q}{\Delta t} = \frac{C \Delta V}{\Delta t}$
    - $\Delta V = \frac{(V_{emf} - V_C)\Delta t}{RC}$